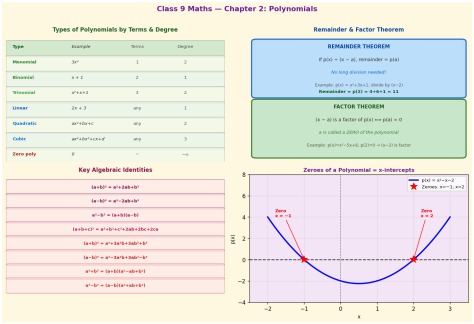
BrainyWorkbook Smart Practice, Sharper Minds

Class 9 **Subject** Mathematics **Chapter** Polynomials **Level** MEDIUM

Name: ..... Date: .....



Polynomials — Class 9 Mathematics, Chapter 2

MEDIUM — Apply the Theory! (Factor, Prove, Compute)

Factorisation	Identities	Proofs & Application
Factor Theorem — find k, factorise	Cubic identities $(a+b)^3$, $(a-b)^3$	Prove Remainder Theorem
Cubic factorisation	$a^3+b^3+c^3-3abc$ identity	Real-world polynomial problems
Sum/difference of cubes	Evaluate without direct calculation	Geometry using polynomial identities

MEDIUM — Think and Apply! (20 Questions)

1 If $(x - 2)$ and $(x + 3)$ are both factors of $p(x) = x^3 + ax^2 + bx - 6$, find the values of a and b. Then factorise p(x) completely.

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2 Factorise each cubic polynomial using the Factor Theorem:

(a) $x^3 - 6x^2 + 11x - 6$

(b) $x^3 + x^2 - 4x - 4$

(c) $2x^3 + x^2 - 13x + 6$

(Hint: Try $x = 1, -1, 2, -2$ as potential zeroes.)

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3 Expand using the cubic identity $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$:

(a) $(x + 2)^3$

(b) $(2x + 1)^3$

(c) $(x + y)^3 + (x - y)^3$ — simplify fully

4 Factorise using sum/difference of cubes ($a^3 + b^3 = (a+b)(a^2-ab+b^2)$, $a^3 - b^3 = (a-b)(a^2+ab+b^2)$):

(a) $8x^3 + 27$

(b) $125 - 64x^3$

(c) $x^3y^3 + 1$

5 *Brainy challenges you!* Evaluate WITHOUT direct computation (use identities):

(a) $(99)^3$ using $(100 - 1)^3$

(b) $(1.02)^3$ using $(1 + 0.02)^3$

(c) $(a + b)^3 - 3ab(a + b)$ — simplify and identify what this equals.

6 If $a + b + c = 0$, prove that $a^3 + b^3 + c^3 = 3abc$.
This uses the identity: $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$.
Now use this result to evaluate: if $x + y + z = 0$, find $(x^3 + y^3 + z^3)/(xyz)$.

7 The volume of a rectangular box is given by $V(x) = x^3 + 6x^2 + 11x + 6$.

(a) Find a factor of $V(x)$ by trying $x = -1$.

(b) Factorise $V(x)$ completely. What do the three factors represent?

(c) If $x = 2$, what are the three dimensions of the box? Verify the volume.

8 Use the identity $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$ to find:

(a) If $a + b + c = 5$ and $a^2 + b^2 + c^2 = 11$, find $ab + bc + ca$.

(b) If $a + b + c = 6$ and $ab + bc + ca = 11$, find $a^2 + b^2 + c^2$.

9 Find the remainder when:

(a) $x^3 + 3x^2 - x + 2$ is divided by $(x - 1/2)$

(b) $x^4 - 2$ is divided by $(x + 1)$

(c) $p(x) = ax^2 + bx + c$ is divided by $(x - 1)$ and $(x + 1)$. Show that the sum of the two remainders is $2p(0) + 2 \cdot$ (coeff of x^2).

10 A farmer's rectangular field has length $(x + 5)$ m and width $(x - 3)$ m.

(a) Write the area as a polynomial in x.

(b) If $x = 10$, what is the area?

(c) If the area must equal 45 m^2, set up and solve a quadratic equation to find x.

(d) Check: are both length and width positive for your value of x?

11 Factorise the following by splitting the middle term:

(a) $6x^2 + 17x + 5$

(b) $4x^2 - 13x + 3$

(c) $2x^2 + 7x - 15$

12 If $p(x) = x^3 - px + q$ has $(x - 1)$ as a factor AND leaves remainder 2 when divided by $(x - 2)$, find p and q.

13 Spot and correct ALL errors in this student's working:
Student's work: "Factorise $x^3 - 8$: $x^3 - 8 = (x - 2)^3$. Expanding: $(x-2)^3 = x^3 - 3(2)x^2 + 3(4)x - 8 = x^3 - 6x^2 + 12x - 8$. So $(x-2)^3 \neq x^3 - 8$. Remainder when x^3+1 is divided by $x+1$: let $x = -1$, remainder = $p(1) = 1+1 = 2$."
Identify all mistakes and provide the correct solution for factorising $x^3 - 8$.

14 Prove: if a polynomial $p(x)$ of degree n has zeroes $a_1, a_2, \ldots, a_n$, then $p(x) = k(x - a_1)(x - a_2)\ldots(x - a_n)$ where k is the leading coefficient. Verify this for $p(x) = 2x^2 - 5x + 3$ (find zeroes, write in factored form, expand, check).

15 Use
the identity $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$ to evaluate:

(a) $1^3 + 2^3 + (-3)^3 - 3(1)(2)(-3) = ?$ (should give 0; verify why)

(b) $(a+b)^3 + (b+c)^3 + (c+a)^3 - 3(a+b)(b+c)(c+a)$ when $a+b+c = 0$.

16 Factorise using algebraic identities:

(a) $27x^3 + y^3 + z^3 - 9xyz$

(b) $x^4 - y^4$ (factorise completely)

(c) $(x^2 + y^2 - z^2)^2 - 4x^2y^2$

17 The area of a square carpet is $(4x^2 + 12x + 9)$ sq. m. Find its side length. If the perimeter must be at most 28 m, what is the maximum value of x?

18 If α and β are zeroes of the quadratic polynomial $p(x) = ax^2 + bx + c$, prove that:

(a) $\alpha + \beta = -b/a$ (sum of zeroes)

(b) $\alpha \cdot \beta = c/a$ (product of zeroes)

(c) Use these to find the sum and product of zeroes of $2x^2 - 5x + 3$ without finding the zeroes explicitly.

19 A ball is thrown upward. Its height (in metres) at time t seconds is given by $h(t) = -5t^2 + 20t + 1$.

(a) What is the height at t = 0? (initial height)

(b) At what time does the ball reach 21 m? (Set h(t) = 21 and solve)

(c) Factorise $-5t^2 + 20t + 1 - 21$ to find when h = 21.

20 Use the identity $(x + a)(x + b) = x^2 + (a+b)x + ab$ to compute:

(a) 103×107 (Hint: = (100+3)(100+7))

(b) 9.8×10.2 (Hint: = (10−0.2)(10+0.2))

(c) 97×103

 Brainy Bonus: The identity $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$ has a beautiful secret: when a+b+c = 0, then $a^3 + b^3 + c^3 = 3abc$ ALWAYS. This is used in competitive mathematics to solve problems like: "If $x = \sqrt[3]{2} - \sqrt[3]{4} + \sqrt[3]{(2/3)}$, find $x^3 + 3x$." Mathematicians use polynomial identities as "shortcuts through calculation space" — what would take pages of algebra collapses to one elegant step. The ancient Indian algebraist Brahmagupta (7th century CE) used polynomial identities extensively in number theory. His "Brahmagupta–Fibonacci identity" is still used in cryptography today!